Starobinsky-Type Inflation from α' - Corrections

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Abstract. Working in the Large Volume Scenario (LVS) of IIB Calabi-Yau flux compactifications, we construct inflationary models from recently computed higher derivative $(\alpha')^3$-corrections. Inflation is driven by a Kähler modulus whose potential arises from the aforementioned corrections, while we use the inclusion of string loop effects just to ensure the existence of a graceful exit when necessary. The effective inflaton potential takes a Starobinsky-type form $V = V_0(1 - e^{-\nu\phi})^2$, where we obtain one set-up with $\nu = -1/\sqrt{3}$ and one with $\nu = 2/\sqrt{3}$ corresponding to inflation occurring for increasing or decreasing ϕ respectively. The inflationary observables are thus in perfect agreement with PLANCK, while the two scenarios remain observationally distinguishable via slightly varying predictions for the tensor-to-scalar ratio r. Both set-ups yield $r \simeq (2\ldots7) \times 10^{-3}$. They hence realise inflation with moderately large fields $(\Delta\phi \sim 6\, M_{Pl})$ without saturating the Lyth bound. Control over higher corrections relies in part on tuning underlying microscopic parameters, and in part on intrinsic suppressions. The intrinsic part of control arises as a leftover from an approximate effective shift symmetry at parametrically large volume.

Contents

1   Introduction

Inflationary cosmology constitutes a rather successful paradigm for solving the conventional hot big bang initial condition problems. The simplest models of inflation create the very early exponentially accelerated expansion via vacuum energy domination of a single slowly rolling scalar field. While inflation enjoys by now robust support from probes of the cosmic microwave background (CMB) such as WMAP, Planck, and BICEP2/Keck Array [1–3], it is in need of UV completion due to its intrinsic sensitivity to higher order corrections to the relevant couplings in the scalar Lagrangian describing the slow-roll process.

One of the historically earliest models of slow-roll inflation arises in the Starobinsky model [4] (for the simplified $R + R^2$ form see also [5]) by changing the Einstein-Hilbert gravitational action to a form

$$S = \int \mathrm{d}^4 x \sqrt{-g} \, \frac{M_{Pl}^2}{2} \left(R + \frac{1}{6M^2} R^2 \right) \quad . \tag{1.1}$$

This is conformally equivalent to a canonically normalised scalar field with an exponentially flat 'plateau' potential

$$V = V_0 \left(1 - e^{-\sqrt{2/3}\phi} \right)^2 \tag{1.2}$$

(and further to a quartic potential for a canonically normalised, but non-minimally coupled scalar field in 'Jordan frame', as e.g. in Higgs inflation [6]). Models of this 'plateau' type have been studied extensively in recent years where they have been found to arise as attractor limits of a large variety of different inflaton potentials in the context of Jordan frame supergravity [7–19]. Moreover, it predicts inflationary observables $n_s = 1 - 2/N \simeq 0.97$ and $r = 12/N^2 \simeq 0.003$ well in agreement with the recent CMB data.

Note, that COBE normalisation of the curvature perturbation requires $V_0 \sim M^2 M_{Pl}^2 \sim 10^{-10} M_{Pl}^4$). This tells us that $M \sim 10^{-5} M_{Pl}$ is small, and thus the R^2 appears strongly enhanced. Hence, positing literally just the semi-infinite plateau potential or, equivalently, the above modified gravitational action would correspond to defining the UV theory of gravity by eq. (1.1).

However, once we look at an actual candidate for a UV theory of quantum gravity, string theory (or even just the supergravity embeddings of Starobinsky-type potentials mentioned above) tells us that the simple UV structure of gravity assumed in eq. (1.1) does not hold. Already the first extant string or supergravity embeddings of such plateau-type moduli inflation models (such as fibre inflation [20] or poly-instanton inflation [21]) show that the pure half-infinitely extended plateau potential acquires a whole series of either exponentially rising or falling corrections at a finite distance in field space which limit the width of the inflationary plateau. Clearly, even when restricting the description of these potentials to the pure Einstein gravity + inflaton scalar field sector, a 4D conformal transformation of these models into the Starobinsky-like frame of eq. (1.1) would show the appearance of $R^n, n \geq 3$ terms (besides all the other non-R^n-type higher curvature terms descending directly from string compactification), see e.g. [22], which invalidate the assumed UV form of eq. (1.1).

Existence of an extended inflationary plateau of width $\Delta\phi \sim 5\,M_{Pl}$ necessary for about 60 e-folds of slow-roll inflation requires an all-order suppression of the prefactors of higher-order rising or falling exponential corrections. Hence, this motivates constructing such exponential plateau-type potentials with plateaus of finite width limited by exponentially rising of falling corrections in UV completions like string theory.

Compactifications of type IIB string theory on Calabi-Yau orientifolds with 3-form flux fixing the complex structure moduli and the axio-dilaton, and a combination of non-perturbative superpotential corrections and $\mathcal{O}(\alpha'^3)$ corrections to the Kähler potential allow for a landscape of moduli-free metastable dS vacua with certain classes of inflationary regions arising from flat potential regions for the Kähler moduli. In a subclass of these models called the Large Volume Scenario (LVS) compactification on a K3-fibred Calabi-Yau produces a Kähler moduli scalar potential which is flat at leading order in the LVS mechanism for the volume modulus τ_1 of the K3-fibre. In 'fibre inflation' [20] the generic form of string loop corrections to the Kähler potential [23, 24] produces a scalar potential with a minimum for the fibre modulus, and upon tuning the string loop coefficients via their complex structure moduli dependence and under certain assumptions about the higher string-loop contributions the 1-loop terms generate an extended exponential plateau potential driving inflation as well. The exponential form the potential terms arises in these models from canonically

normalising the moduli kinetic terms. A different but closely related model is that of poly-instanton inflation [21] where instantonic corrections to instantons are responsible for the genesis of the potential for the fibre modulus. In this case the potential features double exponentials, which make it flatter than Starobinsky-like models, and imply a lower tensor signal. In both of these fibre inflation setups the width of the inflationary plateau is always limited by the appearance of rising exponential terms in the scalar potential from some of the string loop corrections.

In this paper we report on an improved construction of Starobinsky-type plateau potentials in type IIB string compactifications based on the original setup of fibre inflation in [20]. This construction uses the recent discovery [25] of a class of four-superspace-derivative operator contributions to the scalar moduli potential arising from the 10D $\mathcal{O}(\alpha'^3)R^4$ corrections. We can arrange for a trans-Planckian inflationary plateau for the fibre modulus to arise *only* from the scalar potential induced by this new $\mathcal{O}(\alpha'^3)$-contribution and the well known $\mathcal{O}(\alpha'^3)$-correction the volume moduli Kähler potential, with the string loop corrections relegated to the tasks of just providing post-inflationary minima for the fibre modulus.

This provides us with a modicum of control over higher-order corrections, since the inflationary plateau arises from α'-corrections only, and the g_s-suppression of the string loops guarantees an all-order protection from the loops once the 1-loop contributions are small. Control over even higher-order α'-corrections in turn arises from the suppression of these terms at large stabilised volume. This leaves the remaining amount of necessary tuning to appear in the (complex structure moduli VEV dependent) string loop coefficients, which need to be subdominant to the α'-corrections on the inflationary plateau while generating a minimum for the inflaton fibre modulus away from the plateau. Effectively, the smallish-g_s based decoupling of the string loops from the inflationary plateau, and the large-volume control of successively higher-order α'-corrections constitutes a leftover from the approximate Kähler moduli shift symmetry at parametrically large volume.

Dividing the tasks of generating a minimum for the fibre modulus and generating its inflationary plateau between the string loop and α'-corrections this way instead of having one type corrections doing everything leads in addition to a mini-landscape of inflationary regimes and minima – namely it is possible to get inflation rolling both *to the right* and *to the left*. The inflationary potential then takes a form following the discussion above which can be summarised as

$$
V = \begin{cases} V_0 \left(1 - e^{-\nu\phi}\right)^2 + \varepsilon^2 \, e^{\frac{\nu}{2}\phi} + \ldots \text{ rolling to the left} \\[2mm] V_0 \left(1 - e^{\nu\phi}\right)^2 + \varepsilon^2 \, e^{-2\nu\phi} + \ldots \text{ rolling to the right} \end{cases} . \tag{1.3}
$$

A successful construction arranges for $\varepsilon \ll 1$ ensuring the appearance of a finite-width inflationary plateau. For the simple toy K3-fibred geometries we are looking at, we find that the structure of the four-derivative α'-correction [25] generating the plateau region of the potential leads to values for $\nu = 2/\sqrt{3}$, $\nu = 1/\sqrt{3}$ differing between inflation rolling to the left, and to the right, respectively. The predictions for the inflationary

observables hence split as well into $n_s = 1 - 2/N \simeq 0.97$, and $r \simeq 0.002$ vs $r \simeq 0.007$, respectively.

The rest of the paper is structured as follows. We first review the framework of LVS and recall the recent $(\alpha')^3$-corrections to the 4D effective action of type IIB Calabi-Yau-orientifold compactifications with background fluxes in section 2. In section 3 we describe the toy 3-parameter setup of a K3-fibred Calabi-Yau and its Kähler moduli scalar potential arising from the combination of a standard LVS-type stabilisation of two of the three volume modulus and the fibre modulus acquiring a plateau inflation scalar potential with minimum from the combined effect of the string loop and two- and four-derivative $\mathcal{O}(\alpha'^3)$ contributions to the scalar potential. We discuss the various left- and right-rolling branches of this 'breathing fibre inflation', and then move to the extraction of the inflationary observables in section 4. We finally conclude in section 5.

2 Large Volume Limit of IIB Flux compactifications

2.1 The Large Volume Scenario

We will begin by briefly reviewing the framework of LVS. Our starting point is the large volume limit of the low-energy 4D $\mathcal{N} = 1$ - effective action of type IIB Calabi-Yau-orientifold compactifications with background fluxes [26, 27] including the leading order $(\alpha')^3$-corrections to the bulk fields [26] and string-loop corrections [28, 29]. Furthermore, we consider non-perturbative corrections descending from gaugino condensation on wrapped D7-branes or from Euclidean wrapped D3-branes. The background fluxes admit supersymmetric minima for the dilaton as well as complex structure moduli at tree level. After replacing these fields with their respective minima in the effective Lagrangian, the theory is subject to the following Kähler and superpotential

$$K(T + \bar{T}) = \ln(g_s) - 2\ln(\mathcal{V} + \tfrac{1}{2}\hat{\xi}) + \delta K_{g_s}^{KK} + \delta K_{g_s}^{W}$$

$$W(T) = W_0 + \sum_i A_i e^{-a_i T_i} \equiv \frac{1}{\sqrt{2}} e^{\langle K_{cs}\rangle/2} \left(\tilde{W}_0 + \sum_i \tilde{A}_i e^{-a_i T_i} \right) . \tag{2.1}$$

Here K_{cs} denotes the Kähler potential of the complex structure moduli, g_s the string coupling and $\tilde{W}_0$ is the Gukov-Vafa-Witten superpotential evaluated at the supersymmetric minimum.[1] The non-perturbative corrections from wrapped D-branes induce a term in the superpotential. Here $\tilde{A}_i$ and a_i can be regarded as constants, that are related to the specifics of the mechanism which generates these non-perturbative contributions. In order to be able to neglect higher instanton corrections, it is necessary to ensure that $a_i T_i \gg 1$. Moreover, the total volume modulus reads

$$\mathcal{V} = \tfrac{1}{6} k_{ijk} t^i t^j t^k , \tag{2.2}$$

where k_{ijk} denote triple intersection numbers and t^i two-cycle volumes. The latter are implicitly related to the real parts of the Kähler coordinates, which are given by the

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[1]Here we have absorbed the term $\langle K_{cs}\rangle - \ln(2)$ into the superpotential by means of a Kähler transformation.

dual four-cycle volumes via

$$\frac{1}{2}(T_i + \bar{T}_i) \equiv \tau_i = \frac{\partial \mathcal{V}}{\partial t^i} = \tfrac{1}{2}k_{ijk}t^j t^k \ . \tag{2.3}$$

Furthermore, $\hat{\xi}$ parametrises the leading order $(\alpha')^3$-corrections and reads [26]

$$\hat{\xi} = -\frac{(\alpha')^3 \zeta(3)\chi(M_3)}{2(2\pi)^3 g_s^{3/2}} \ , \tag{2.4}$$

where $\chi(M_3)$ denotes the Euler number of the threefold. Lastly, the Kähler potential includes string-loop corrections which are induced from the exchange of closed strings carrying Kaluza-Klein momentum as well as from the exchange of winding strings. Their general form was inferred in [29] to be

$$\delta K_{(g_s)}^{KK} \sim g_s \sum_{i=1}^{h^{1,1}} \frac{C_i^{KK}(a_{ij}t^j)}{\mathcal{V}} \ , \qquad \delta K_{(g_s)}^{W} \sim \sum_{i=1}^{h^{1,1}} \frac{C_i^{W}(a_{ij}t^j)^{-1}}{\mathcal{V}} \ . \tag{2.5}$$

Here C_i^{KK} and C_i^W can be regarded as constants, which depend on the complex structure moduli at the minimum, and a_{ij} are combinatorial constants. For the particular example of $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ the explicit form of C_i^{KK} and C_i^W was computed in [28]. For this example it was shown in [30] that as long as the complex structure moduli are not stabilised large, and note that stabilising the complex structure moduli at large values would include a certain amount of fine-tuning, one finds that roughly

$$C_i^{KK} \simeq C_i^W \simeq \frac{1}{128\pi^4} \ , \tag{2.6}$$

which in the following will serve as a ballpark estimate of these constants. The scalar potential derived from (2.1) can be expanded in the large volume limit in inverse powers of $\mathcal{V}$. The leading order contributions from the individual terms (2.1) read

$$V = V^{LVS} + \delta V_{(g_s)} \equiv V_{np} + V_{\alpha'} + \delta V_{(g_s)} \ , \tag{2.7}$$

where [23, 24]

$$V_{np} = e^K K_0^{i\bar{j}} \left(a_i a_j A_i \bar{A}_{\bar{j}} e^{-(a_i T_i + a_j \bar{T}_{\bar{j}})} - a_i A_i e^{-a_i T_i} \bar{W} K_{0,\bar{j}} - a_j \bar{A}_{\bar{j}} e^{-a_j \bar{T}_{\bar{j}}} W K_{0,i} \right)$$

$$V_{\alpha'} = g_s \frac{3\hat{\xi}|W_0|^2}{4\mathcal{V}^3} \ , \qquad \delta V_{(g_s)} = \sum_i \frac{g_s|W_0|^2}{\mathcal{V}^2} \left[g_s^2 (C_i^{KK})^2 K_{(0),ii} - 2\delta K_{(g_s),t^i}^W \right] \ . \tag{2.8}$$

Here $K_0 = -2\ln\mathcal{V}$ and we have assumed that the D-branes only wrap basis four-cycles. To achieve a large stabilised volume additional assumptions regarding the triple intersection numbers have to be made. In the original model in [31] the volume was assumed to be controlled by a single four-cycle and that furthermore a blow-up cycle exists, for which non-perturbative corrections to W are present. Then V^{LVS} has a controlled minimum with exponentially large volume. The string-loop correction $\delta V_{(g_s)}$ is particularly interesting for those compactification-geometries for which V^{LVS} has a flat direction or no minimum at large volume exists. A particular case, in which V^{LVS} has an exact flat direction was studied in [20, 24].

2.2 Four-Superspace-Derivative Correction

Besides the $(\alpha')^3$-correction to the Kähler potential in (2.1) further $(\alpha')^3$-corrections are expected to enter the 4D effective action of type IIB Calabi-Yau-orientifold compactifications with background fluxes. It was argued recently in [25] that these corrections have to be described off-shell via higher-derivative terms in $\mathcal{N} = 1$ - superspace. Moreover, in [25] the leading order operator in the higher-superspace-derivative expansion was computed by KK-reducing $(\alpha')^3$-corrections in 10D. At order $(\alpha')^3$ this operator contributes four-derivative and additional two-derivative-terms as well as a new term in the scalar potential to the effective action of the Kähler moduli. Even though the derivative-terms are not known exactly, the functional form of the correction to the scalar potential could be inferred by using the no-scale property of K_0 and reads[2]

$$V_{(1)} = -g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \Pi_i t^i \,, \tag{2.9}$$

where $\hat{\lambda} = \lambda (\alpha')^3 g_s^{-3/2}$ with λ being an undetermined combinatorial constant. From the 10D origin of the correction one can infer the rough estimate

$$|\hat{\lambda}| \simeq \left| \frac{\hat{\xi}}{\chi(M_3)} \right| \,, \qquad \text{and hence} \qquad |\lambda| \simeq \frac{\zeta(3)}{2(2\pi)^3} \,. \tag{2.10}$$

Furthermore, the Π_i in (2.9) are integer numbers encoding the topological information of the second Chern class $c_2(M_3)$. More precisely, upon choosing $\hat{D}_i$ as a basis of harmonic $(1, 1)$-forms on M_3, such that the Kähler form reads

$$J = \sum_{i=1}^{h^{1,1}} \hat{D}_i t^i \,, \tag{2.11}$$

we have

$$\Pi_i = \int_{M_3} c_2 \wedge \hat{D}_i \,. \tag{2.12}$$

If the two-cycles were chosen such that (2.11) holds, then they span the Kähler-cone and automatically $t^i \geq 0$ independently for each $i = 1, \ldots, h^{1,1}$. This in turn implies that $\Pi_i \geq 0$ for each $i = 1, \ldots, h^{1,1}$. With respect to an arbitrary choice of two-cycles the numbers Π_i can have both signs.

3 Towards an Inflationary Regime

3.1 Volume Stabilisation for $K3$-Fibered Threefold

We will consider the compactification geometry investigated in [20, 24] for which a large-volume scenario with a flat direction exists (that is without including $V_{(1)}$ or

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[2]Strictly speaking this formula is only valid for models without non-perturbative corrections to W. However, we will consider models where $V_{(1)}$ is the relevant potential for those fields which are flat directions of V^{LVS} and for which non-perturbative corrections are either absent or negligible so that indeed the form of $V_{(1)}$ is correct. Blow-ups and the volume modulus are stabilised by V^{LVS} and are approximately unaffected by $V_{(1)}$.

$\delta V_{(g_s)}$). More precisely, this geometry can be regarded as "adding a blow-up cycle" to the $K3$-fibered threefold $\mathbb{CP}^4[1,1,2,2,6]$ which has $h^{1,1} = 2$. The volume in terms of the four-cycles is assumed to be of the form

$$\mathcal{V} = \alpha \left(\sqrt{\tau_1}\tau_2 - \gamma\tau_3^{3/2} \right) , \tag{3.1}$$

where τ_1 is associated with the volume of the $K3$-fibre, τ_2 controls the overall volume and τ_3 denotes the blow-up. It is possible to find intersection numbers, which yield a volume as in (3.1) [20]. For instance choosing

$$\mathcal{V} = \lambda_1 t^1 (t^2)^2 + \lambda_3 (t^3)^3 , \tag{3.2}$$

we find

$$\tau_1 = \lambda_1 (t^2)^2 , \quad \tau_2 = 2\lambda_1 t^1 t^2 , \quad \tau_3 = 3\lambda_3 (t^3)^2 . \tag{3.3}$$

We will now assume that we are not in the Kähler cone, but that we have chosen a basis such that $t^3 \leq 0$. Inverting (3.3) yields

$$t^1 = \frac{\tau_2}{2\sqrt{\lambda_1 \tau_1}} , \quad t^2 = \sqrt{\frac{\tau_1}{\lambda_1}} , \quad t^3 = -\sqrt{\frac{\tau_3}{3\lambda_3}} . \tag{3.4}$$

Hence, one indeed obtains (3.1) with $\alpha = \frac{1}{2}\lambda_1^{-1/2}$ and $\gamma = \sqrt{\frac{4}{27}}\sqrt{\lambda_1}\lambda_3^{-3/2}$. Let us now briefly summarise the volume stabilisation for (3.1) following [20]. Firstly, the limit of large volume requires $\tau_1, \tau_2 \gg \tau_3$ and, thus, the only relevant terms in the superpotential are given by

$$W(T) \simeq W_0 + A_3 e^{-a_3 T_3} . \tag{3.5}$$

Calculating the Kähler metric and minimising the axionic component of T_3 one obtains [20]

$$V^{LVS}(\mathcal{V}, \tau_3) = g_s \left[\frac{8a_3^2 A_3^2}{3\alpha\gamma} \frac{\sqrt{\tau_3}}{\mathcal{V}} e^{-2a_3\tau_3} - 4W_0 a_3 A_3 \frac{\tau_3}{\mathcal{V}^2} e^{-a_3\tau_3} + \frac{3\hat{\xi}W_0^2}{4\mathcal{V}^3} \right] . \tag{3.6}$$

The above potential indeed does not depend on τ_1, which is, hence, left as a flat direction. V^{LVS} has a minimum at exponentially large volume given by

$$\langle \tau_3 \rangle = \left(\frac{\hat{\xi}}{2\alpha\gamma} \right)^{2/3} , \quad \langle \mathcal{V} \rangle = \frac{3\alpha\gamma}{4a_3 A_3} W_0 \sqrt{\langle \tau_3 \rangle} e^{a_3\langle \tau_3 \rangle} . \tag{3.7}$$

This minimum is expected to be perturbatively stable as long as no unnaturally large coefficients of higher-order operators occur [32]. In the following we assume that τ_3 and $\mathcal{V}$ are sitting at the minimum (3.7) and turn our attention fully to the flat direction τ_1, for which additional perturbative corrections, that is both $\delta V_{(g_s)}$ as well as $V_{(1)}$, are important. Moreover, as the size of these corrections is sub-leading compared to V^{LVS} we naively expect τ_1 to be the lightest modulus and, hence, τ_1 possibly constitutes a candidate to drive inflation.

3.2 Perturbative Corrections and Inflation

We begin the analysis of additional corrections to the scalar potential by the higher-derivative $(\alpha')^3$-correction given in (2.9). For the geometry in defined in (3.1) and (3.4) $V_{(1)}$ reads

$$V_{(1)} \simeq -g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \left(\Pi_1 \frac{\mathcal{V}}{\tau_1} + \Pi_2 \lambda_1^{-1/2} \sqrt{\tau_1} \right) \ . \tag{3.8}$$

Note that we neglected all τ_3-dependent terms in 3.8. In what follows we will ignore such terms as they play no role in the phenomenology we are interested in here.

After replacing $\mathcal{V}$ and τ_3 with their values at the LVS minimum (3.7) the scalar potential, excluding the string-loop corrections for the moment, is of the form

$$V(\tau_1) = V^{LVS}(\langle\tau_3\rangle, \langle\mathcal{V}\rangle) - g_s^2 \hat{\lambda} \frac{|W_0|^4}{\langle\mathcal{V}\rangle^4} \left(\Pi_1 \frac{\langle\mathcal{V}\rangle}{\tau_1} + \Pi_2 \lambda_1^{-1/2} \sqrt{\tau_1} \right) \ . \tag{3.9}$$

In the case $\Pi_1, \Pi_2 > 0$ and $\hat{\lambda} < 0$ this potential features a minimum. Slow roll inflation can in principle occur around this minimum. It is convenient to discuss inflation in terms of a canonically normalised field. The normalisation can be performed following [20]. After replacing $\tau_2 = \tau_2(\tau_1, \mathcal{V}, \tau_3)$ via (3.1) in the Lagrangian, the kinetic terms read

$$\mathcal{L} \supset -\frac{3}{8\tau_1^2}(\partial_\mu \tau_1 \partial^\mu \tau_1) + \frac{1}{2\tau_1 \mathcal{V}}(\partial_\mu \tau_1 \partial^\mu \mathcal{V}) - \frac{1}{2\mathcal{V}^2}(\partial_\mu \mathcal{V} \partial^\mu \mathcal{V}) + \dots \tag{3.10}$$

Thus, the canonically normalised inflaton φ is related to τ_1 as follows

$$\tau_1 = e^{2\varphi/\sqrt{3}} \ . \tag{3.11}$$

In this formula τ_3-dependent corrections were neglected. In [20] these have already been argued to be negligible. Moreover, the kinetic terms obtain $(\alpha')^3$-corrections both from the $\hat{\xi}$-term in the Kähler potential in (2.1) as well as the higher superspace-derivative correction [25]. For the former correction the canonical normalisation is altered schematically at leading order in α' as

$$\tau_1 \simeq e^{2\varphi/\sqrt{3}} \left(1 + \hat{\xi} \frac{\varphi}{\mathcal{V}} + \mathcal{O}(\alpha'^6) \right) \ . \tag{3.12}$$

Such corrections are negligible as long as $\varphi \ll \mathcal{V}/\hat{\xi}$. Unfortunately the higher superspace-derivative correction to the two-derivative term is not exactly known. Nevertheless, we can give an estimate based on the approximate correction [25]

$$\mathcal{L} \supset g_s \hat{\lambda} |W_0|^2 \frac{\Pi_l t^l}{\mathcal{V}^2} K_{0,i} K_{0,j} \left(\partial_\mu \tau_i \, \partial^\mu \tau_j \right) \ . \tag{3.13}$$

Using (3.1) and (3.4) we again calculate the leading order correction to the canonical normalisation, this yields

$$\tau_1 \simeq e^{2\varphi/\sqrt{3}} + g_s \hat{\lambda} |W_0|^2 \left(\frac{\Pi_1}{\mathcal{V}} + \frac{\Pi_2}{\mathcal{V}^2} e^{\sqrt{3}\varphi} \right) + \mathcal{O}(\alpha'^6) \ . \tag{3.14}$$

The corrections are again numerically suppressed due to the appearance of $\hat{\lambda}$ and inverse volume factors. The last correction will, however, become important for $\varphi \gg 1$. In the following we will be interested in an intermediate φ-regime, where we can safely neglect the above corrections.

Using (3.11) we can rewrite the scalar potential as

$$V(\varphi) = V^{LVS}(\langle \tau_3 \rangle, \langle \mathcal{V} \rangle) - g_s^2 \hat{\lambda} \frac{|W_0|^4}{\langle \mathcal{V} \rangle^4} \left(\Pi_1 \langle \mathcal{V} \rangle e^{-2/\sqrt{3}\varphi} + \Pi_2 \lambda_1^{-1/2} e^{\varphi/\sqrt{3}} \right). \qquad (3.15)$$

Inflation driven by a single rising exponential $V \sim e^{\nu\varphi}$ has $n_s = 1 - \nu^2$ and $r = 8\nu^2$ and is therefore well outside the preferred region in the (n_s, r) plane [2] with $\nu = 1/\sqrt{3}$ or $\nu = -2/\sqrt{3}$. So in this case even though the lightest modulus, τ_1, is stabilised by the $(\alpha')^3$-corrections and the resulting potential can lead to a long lasting period of inflation, its observational signatures are essentially ruled out by current data.

However, while τ_1 is the lightest modulus which renders the leading-order dynamics single-field, the remaining heavier moduli will eventually backreact due to the inflationary vacuum energy built up by displacing τ_1. This eventual backreaction will try to lower the overall vacuum energy during inflation and will hence lead to flattening [33] of the effective inflaton potential. We leave it for future work to determine if this effect applied to exponential scalar potentials is strong enough to lower r and shift n_s sufficiently to acquire observational compatibility for exponential field dependence with $\mathcal{O}(1)$ coefficients in the exponents.

One may alternatively consider geometries with $\Pi_1 < 0 \wedge \Pi_2 > 0$ or $\Pi_1 > 0 \wedge \Pi_2 < 0$. The resulting potential will feature an extended plateau where inflation may occur. Yet in this regime the potential of eq. (3.9) does not have a local minimum and is furthermore unbounded from below. The various possibilities are illustrated in figure 1. With both Π_i negative, the potential becomes a hilltop where inflation may occur on both sides, yet the problem of the potential not being bounded from below remains.

In order to obtain sensible inflationary scenarios with a viable exit we now turn our attention to string-loop corrections.

3.3 Minima from String Loops

The string-loop corrections to the scalar potential (2.8) for the compactification geometry (3.1), (3.4) take the form [20]

$$\delta V_{(g_s)} \simeq \frac{g_s |W_0|^2}{\mathcal{V}^2} \left(g_s^2 \frac{(C_1^{KK})^2}{\tau_1^2} + 2g_s^2 (\alpha C_2^{KK})^2 \frac{\tau_1}{\mathcal{V}^2} \right). \qquad (3.16)$$

Here we neglected the KK contribution with respect to τ_3, since it carries no dependence on τ_1 and is, thus, not relevant for our discussion. Moreover, note that the winding-mode contributions to $\delta V_{(g_s)}$ are induced by strings which wind one-cycles on the intersection locus of stacks of $D7$-branes. Such one-cycles can exist in the intersection of four-cycles, which the $D7$-branes wrap. However, since the four-cycle associated with τ_3 only intersects with itself, the only winding contribution to $\delta V_{(g_s)}$ that can be induced is via an intersection of the four-cycles associated with τ_1 and τ_2. In the
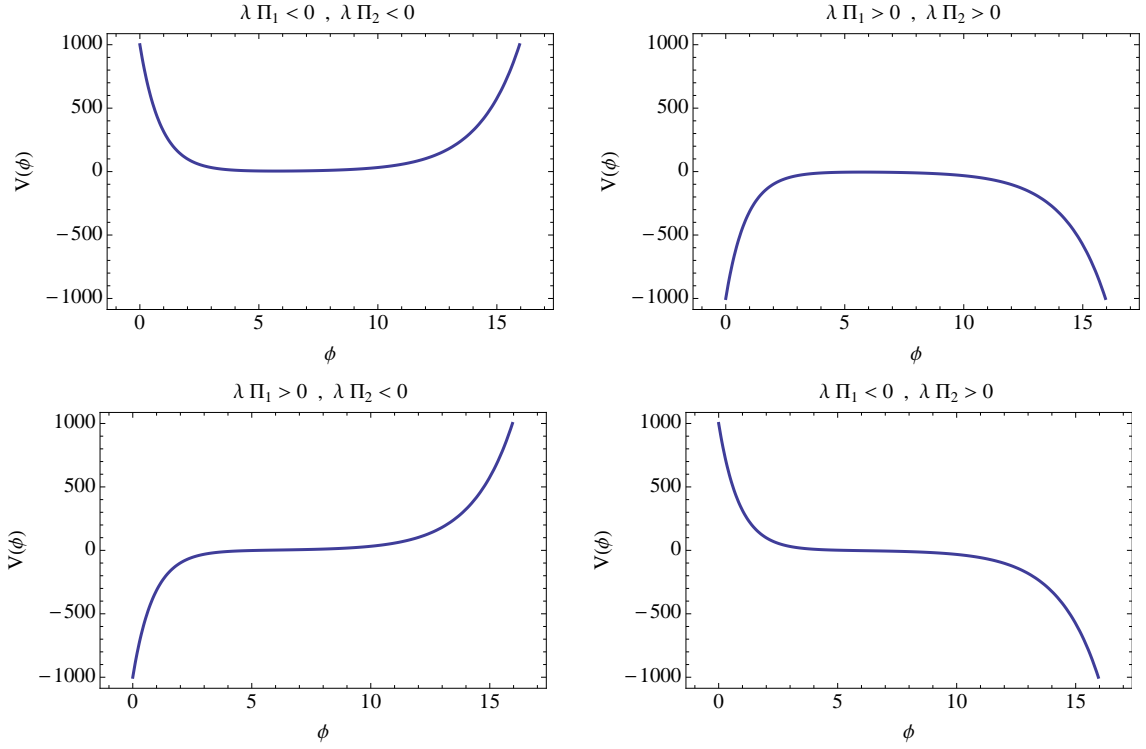
Figure 1. *Scalar potential of Eq. (3.15) (in units of $g_s^2 \left(\frac{W_0}{\mathcal{V}}\right)^4$) for different choices of the topological integers Π_1 and Π_2.*

following for simplicity we consider configurations where the $D7$-branes only wrap those four-cycles which are associated with τ_2 or τ_3 and hence winding-mode contributions are completely absent. Note that, in principle we need not assume any special brane-configuration as long as the coefficient of the winding-mode string-loop correction is tuned small enough.

The string-loop corrections in (3.16) are suppressed with additional powers of $g_s^{5/2}$ with respect to $V_{(1)}$ in (3.8). Moreover, noting that the typical size of the topological numbers was inferred in [25] to be $\Pi_i \sim \mathcal{O}(10\ldots100)$ and using the estimates (2.6) and (2.10) we find that

$$|C_1^{KK}|^2 \sim |C_2^{KK}|^2 \ll |\lambda||\Pi_i| \ . \tag{3.17}$$

Thus, for moderately small $g_s \lesssim 10^{-1}$ and $W_0 \gtrsim 1$ the string-loop corrections are suppressed compared to $V_{(1)}$ in some regime of τ_1. However, for small enough and large enough τ_1 the contributions $\delta V_{(g_s),1}^{KK}$ and $\delta V_{(g_s),2}^{KK}$ will become important and eventually dominate over the terms in $V_{(1)}$. Since $\delta V_{(g_s),1}^{KK}$ and $\delta V_{(g_s),2}^{KK}$ are strictly positive, one may use them to stabilise the fibre modulus. In the intermediate τ_1 regime they will remain subleading compared to $V_{(1)}$. In this way one has a scenario where inflation originates from the higher derivative corrections to the scalar potential while a graceful exit results from the interplay between these corrections and string loops.

The full scalar potential, that is taking into account string loop-corrections and

$V_{(1)}$, is now bounded from below and takes the form

$$
\begin{aligned}
V(\tau_1) &= V^{LVS}(\langle\tau_3\rangle, \langle\mathcal{V}\rangle) + V_{(1)} + \delta V_{(g_s)} \\
&= V^{LVS}(\langle\tau_3\rangle, \langle\mathcal{V}\rangle) - g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \left(\Pi_1 \frac{\mathcal{V}}{\tau_1} + \Pi_2 \lambda_1^{-1/2} \sqrt{\tau_1} \right) \\
&\quad + \frac{g_s |W_0|^2}{\mathcal{V}^2} \left(g_s^2 \frac{(C_1^{KK})^2}{\tau_1^2} + 2 g_s^2 (\alpha C_2^{KK})^2 \frac{\tau_1}{\mathcal{V}^2} \right) + \frac{\epsilon}{\mathcal{V}^2} \,,
\end{aligned}
\tag{3.18}
$$

where an uplift term $\epsilon/\mathcal{V}^2$ has been added.[3] Upon canonical normalisation V can be recast as

$$
V(\varphi) = V^{LVS} + V_0 \left(-\mathcal{C}_1 e^{-2/\sqrt{3}\varphi} - \mathcal{C}_2 e^{\varphi/\sqrt{3}} + \mathcal{C}_1^{loop} e^{-4/\sqrt{3}\varphi} + \mathcal{C}_2^{loop} e^{2\sqrt{3}\varphi} \right) + \delta_{up}, \tag{3.19}
$$

where we have defined

$$
V_0 = g_s^2 \frac{|W_0|^4}{\mathcal{V}^4} \,, \quad \mathcal{C}_1 = \hat{\lambda} \Pi_1 \mathcal{V} \,, \quad \mathcal{C}_2 = \hat{\lambda} \Pi_2 \lambda_1^{-1/2} \,,
$$

$$
\mathcal{C}_1^{loop} = \frac{\mathcal{V}^2}{|W_0|^2} g_s (C_1^{KK})^2 > 0 \,, \quad \mathcal{C}_2^{loop} = \frac{2g_s}{|W_0|^2} (\alpha C_2^{KK})^2 > 0 \tag{3.20}
$$

and $\delta_{up} = \epsilon/\mathcal{V}^2$. We will now tend to the inflationary dynamics in more detail.

3.4 Inflationary Dynamics

Since $\mathcal{C}_1^{loop}, \mathcal{C}_2^{loop} > 0$, i.e. the string loop corrections will ensure that the potential is bounded from below, the resulting inflationary dynamics depend on the signs of the $\mathcal{C}_i$. Being initially placed on a suitable side of the minimum, the inflaton can either inflate to the left, the right, or to both sides (hill-top) given appropriate signs of the exponentials coefficients. For both $\mathcal{C}_i$ negative, the potential will always be dominated by rising exponentials, hence no viable observationally signature appears;

	left	right	hill-top	exp
$\mathcal{C}_1$	> 0	< 0	> 0	< 0
$\mathcal{C}_2$	< 0	> 0	> 0	< 0

Recalling the relevant terms of the potential during inflation

$$
V_{inf} \sim -\frac{\mathcal{C}_1}{\tau_1} - \mathcal{C}_2 \sqrt{\tau_1} \,. \tag{3.21}
$$

To ensure a phenomenologically viable inflationary regime, slow roll should be effectively driven by only one of these terms. Thus, we want the observable ~ 60 e-folds of

---

[3]Positive vacuum energy contributions of the form $\epsilon/\mathcal{V}^p$, $p = 1 \ldots 3$ can arise in type IIB string compactifications from various sources involving moduli and/or matter F- and/or D-terms, as well as certain anti-branes at the tip of warped throats, see e.g. the discussion on pp. 134 in [34] for an overview with literature.
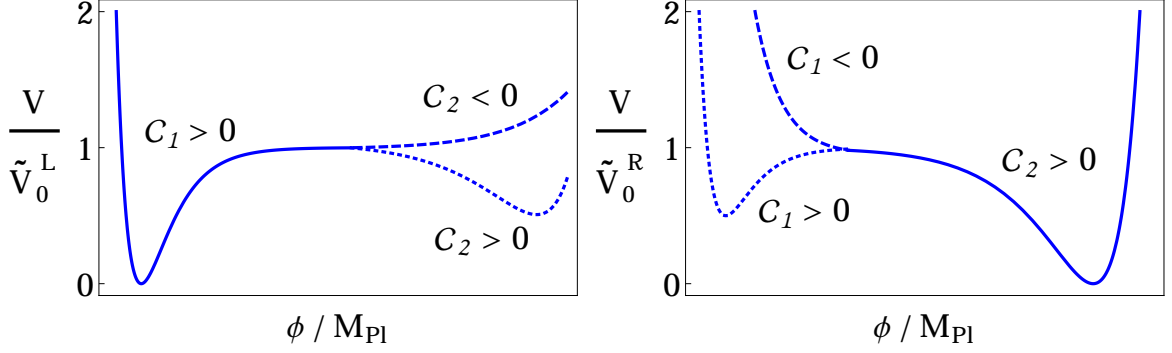
Figure 2. **Left:** *The canonical inflaton potential for $\mathcal{C}_1 > 0$ and arbitrary $\mathcal{C}_2$. String loops ensure that the potential is bounded from below while inflation is driven by the $\mathcal{C}_1$-term of $V_{(1)}$. If $\mathcal{C}_2 > 0$ there can be observed running of the spectral index n_s.* **Right:** *The inflaton rolls to the right. Inflation is driven by the $\mathcal{C}_2$-term of $V_{(1)}$.*

inflation to occur before both terms in V_{inf} become of the same order. This happens for values

$$\tau_1^c \sim \left(\frac{|\mathcal{C}_1|}{|\mathcal{C}_2|}\right)^{2/3} . \tag{3.22}$$

Now, depending on whether inflation is occurring by the inflaton rolling to the left or right as depicted in figure 2, we have to ensure that the plateau appears around values $\tau_1^c > \tau_1$ or $\tau_1^c < \tau_1$. In order to have the potential sufficiently flat, there are bounds on the $\mathcal{C}_i$ and $\mathcal{C}_i^{loop}$ coming from the running of the spectral index, which have to be obeyed when considering viable models of inflating to the right or the left. These are subject of subsection 4.1 and will be found in expression (4.14) and (4.18) respectively.

Inflation to the Right

Let us first consider the case of $\mathcal{C}_2 > 0$ and let us take the $\mathcal{C}_2^{loop}$-term to ensure a minimum; inflation occurs when the field is initially placed on the left side of the minimum and then rolls to the right. This corresponds to an increase of the fibre modulus at constant compactification volume. The inflationary potential is hence given by

$$V_{inf}^R \sim V_0 \left(-\mathcal{C}_2\sqrt{\tau_1} + \mathcal{C}_2^{loop}\tau_1\right), \quad \tau_1 > \tau_1^c. \tag{3.23}$$

We find a minimum

$$\left.\frac{\partial V_{inf}^R}{\partial \tau_1}\right|_{\tau_1 > \tau_1^c} \overset{!}{=} 0 \quad \Rightarrow \quad \tau_1^{min} = \left(\frac{\mathcal{C}_2}{2\,\mathcal{C}_2^{loop}}\right)^2, \tag{3.24}$$

and hence

$$V_{inf}^R(\tau_1^{min}) = -V_0 \frac{\mathcal{C}_2^2}{4\,\mathcal{C}_2^{loop}} \equiv -\tilde{V}_0^R. \tag{3.25}$$

– 12 –

The Minkowski uplifted potential then becomes

$$V_{inf}^R = \tilde{V}_0^R \left(1 - \beta_R \sqrt{\tau_1} + \frac{\beta_R^2}{4} \tau_1 \right)$$

$$= \tilde{V}_0^R \left(1 - \frac{\beta_R}{2} \sqrt{\tau_1} \right)^2, \quad \beta_R \equiv \frac{4\,\mathcal{C}_2^{loop}}{\mathcal{C}_2}. \tag{3.26}$$

Note how the above closes to a square relation. In order to have a sufficient plateau, we require $\beta_R \sqrt{\tau_1} \ll 1 \leftrightarrow \tau_1 \ll \beta_R^{-2}$, i.e. we require $\beta_R \ll 1$ to have a plateau at $\tau_1 \gtrsim 1$. Recalling expressions (3.20) for $\mathcal{C}_2$ and $\mathcal{C}_2^{loop}$, we see that

$$\beta_R = 2\,\frac{g_s^{5/2}(C_2^{KK})^2}{\lambda\,|W_0|^2\Pi_2}, \tag{3.27}$$

which may satisfy $\beta_R \ll 1$ and hence ensures an inflationary plateau. Further, as we want the minimum to be on the right side of τ_1^c in (3.22), we require

$$\tau_1^{min} > \tau_1^c \quad \Rightarrow \quad \mathcal{C}_2^{loop} < \left(\frac{\mathcal{C}_2^4}{|\mathcal{C}_1|} \right)^{1/3} \tag{3.28}$$

which can easily be satisfied using (3.17).

Turning to canonical variables via (3.11), we write potential (3.26) as

$$V_{inf}^R(\varphi) = \tilde{V}_0^R \left(1 - \frac{\beta_R}{2} e^{\frac{\kappa}{2}\varphi} \right)^2 \tag{3.29}$$

with $\kappa = 2/\sqrt{3}$. Shifting this by the vacuum expectation value $\phi = \varphi + 2\,\kappa^{-1}\log(\beta_R/2)$, we arrive at the effective inflaton potential

$$V_{inf}^R(\phi) = \tilde{V}_0^R \left(1 - e^{\frac{\kappa}{2}\phi} \right)^2. \tag{3.30}$$

Finally let us consider the bounds on our parameters arising from the normalisation of the scalar density perturbations [2]

$$\frac{1}{8\pi^2} \frac{H^2}{\epsilon_V} \bigg|_* \sim 2.2 \times 10^{-9}, \tag{3.31}$$

where the star denotes evaluation at horizon exit which we assume to occur 55 e-folds before the end of inflation. Taking into account the correct normalisation of the potential in the 4D Einstein frame, which brings about an extra $1/8\pi$ factor into V [32], this translates into

$$\left(\frac{1}{8\pi} \frac{V_{inf}}{\epsilon_V} \right)^{1/4} \bigg|_{N_e=55} = 0.027\,M_{Pl} \tag{3.32}$$

where ϵ_V is the potential slow-roll parameter which is independent of the normalisation of the potential. Thus, when inflating to the right, we have

$$\tilde{V}_0^R = 8\pi\,0.027^4\,M_P^4\,\epsilon_V\big|_{N_e=55} = 5.7 \times 10^{-9}, \tag{3.33}$$

where we have set $M_P = 1$ in the last equality. This hence sets

$$\tilde{V}_0^R \equiv \frac{g_s^2 |W_0|^4}{\mathcal{V}^4} \frac{\mathcal{C}_2^2}{4 \mathcal{C}_2^{loop}} \sim \lambda^2 \frac{|W_0|^6}{\mathcal{V}^4} g_s^{-2} (C_2^{KK})^{-2} \overset{!}{=} 5.7 \times 10^{-9} \qquad (3.34)$$

which is in accordance with natural choices for g_s, $\mathcal{V}$ and W_0.

We may hence summarise the demands and resulting bounds for a viable inflationary regime as follows

demand	resulting bound
plateau at $\tau_1 \gtrsim 1$	$\beta_R \ll 1$
$\tau_1^{min} > \tau_1^c$	$\mathcal{C}_2^{loop} < \left(\frac{\mathcal{C}_2^4}{|\mathcal{C}_1|} \right)^{1/3}$
COBE	$\lambda^2 |W_0|^6 \mathcal{V}^{-4} g_s^{-2} (C_2^{KK})^{-2} \sim 5 \times 10^{-9}$

If these requirements are met, inflation is fully captured and described by potential (3.30).

Inflation to the Left

Let us now turn to the case $\mathcal{C}_1 > 0$. We ensure the existence of a minimum by considering the $\mathcal{C}_1^{loop}$-term and hence write the inflationary potential as

$$V_{inf}^L = V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} + \frac{\mathcal{C}_1^{loop}}{\tau_1^2} \right). \qquad (3.35)$$

Being initially placed on the right side of the minimum, inflation now occurs when the inflaton rolls to the left. Geometrically this corresponds to a shrinking fibre and a constant overall volume. The minimum of the potential is now found to lie at

$$\tau_1^{min} = \frac{2 \mathcal{C}_1^{loop}}{\mathcal{C}_1}, \qquad (3.36)$$

and the value of the potential at the minimum becomes

$$V_{inf}^L(\tau_1^{min}) = -V_0 \frac{\mathcal{C}_1^2}{4 \mathcal{C}_1^{loop}} \equiv -\tilde{V}_0^L. \qquad (3.37)$$

To be able to safely neglect non-perturbative correction for the Fibre modulus we require that

$$\tau_1^{min} \gtrsim 1 . \qquad (3.38)$$

The Minkowski uplifted potential reads

$$\begin{aligned} V_{inf}^L &= \tilde{V}_0^L \left(1 - \frac{\beta_L}{\tau_1} + \frac{\beta_L^2}{4\tau_1} \right) \\ &= \tilde{V}_0^L \left(1 - \frac{\beta_L}{2\tau_1} \right)^2 , \quad \beta_L = \frac{4 \mathcal{C}_1^{loop}}{\mathcal{C}_1} . \end{aligned} \qquad (3.39)$$

Again, the expression closes to a square. We now have $\tau_1^{min} = \beta_L/2$. hence we require $\beta_L \gtrsim 1$ to keep control over the theory. Recall (3.20) and observe that

$$\beta_L = 4 \frac{\mathcal{V}|W_0|^{-2}g_s^{5/2}(C_1^{KK})^2}{\lambda\,\Pi_1} \tag{3.40}$$

which using (3.17) and given reasonable choices for $\mathcal{V}, g_s$ and W_0 guarantees $\beta_L \gtrsim 1$. Also, as inflation now occurs for the field rolling to the left, we seek

$$\tau_1^{min} < \tau_1^c \quad \Rightarrow \quad \mathcal{C}_1^{loop} < \frac{\mathcal{C}_1^{5/3}}{2\,\mathcal{C}_2^{2/3}}\,, \tag{3.41}$$

which is easily fulfilled using (3.17) and $\mathcal{V} \gg 1$.

Using the canonical normalisation, we rewrite (3.39) as

$$V_{inf}^L = \tilde{V}_0^L \left(1 - \frac{\beta_L}{2}e^{-\kappa\varphi}\right)^2$$
$$= \tilde{V}_0^L \left(1 - e^{-\kappa\phi}\right)^2\,, \tag{3.42}$$

where we have shifted the field φ to $\phi = \varphi - \kappa^{-1}\log(\beta_L/2)$ with $\kappa = 2/\sqrt{3}$.

Again turning to the normalisation of the curvature perturbations (3.32), we find the condition

$$\tilde{V}_0^L = 8\pi\,0.027^4\,M_P^4\,\epsilon_V\big|_{N_e=55} = 1.5 \cdot 10^{-9}. \tag{3.43}$$

or equivalently

$$\tilde{V}_0^L \equiv \frac{g_s^2|W_0|^4}{\mathcal{V}^4}\frac{\mathcal{C}_1^2}{4\,\mathcal{C}_1^{loop}} \sim \lambda^2\frac{|W_0|^4}{\mathcal{V}^4}|W_0|^2 g_s^{-2}(C_1^{KK})^{-2} \overset{!}{=} 1.5 \cdot 10^{-9} \tag{3.44}$$

which may be satisfied given reasonable choices of the involved parameters.

We thus summarise the demands and resulting bounds for a viable inflationary regime as follows

demand	resulting bound
minimum at $\tau_1 \gtrsim 1$	$\beta_L \sim g_s^{5/2}\mathcal{V}(C_1^{KK})^2 \gtrsim 1$
$\tau_1^{min} < \tau_1^c$	$\mathcal{C}_1^{loop} < \frac{1}{2}\left(\frac{2}{\mathcal{C}_2}\right)^{2/3}\mathcal{C}_1^{5/3}$
COBE	$\lambda^2\,|W_0|^6\,\mathcal{V}^{-4}g_s^{-2}(C_1^{KK})^{-2} \sim 10^{-9}$

3.5 Mass Hierarchy

Up to this point our analysis has focused mostly on the fibre modulus and on the constraints that must be met in order to have a simple single field model with the inflaton rolling either to the left or to the right, towards its minimum. Being a string inspired model, there are however plenty more scalars around, describing various aspects of the geometry of the compactification and the position of branes in the extra dimensions.

In order for the above analysis to hold and for this model to be a genuinely single field inflation model, all the remaining scalars must play no role in the inflationary dynamics. The easiest way to achieve this is for them to be parametrically heavier than the Hubble scale during inflation. The hierarchy in the potential between the large volume part and the inflationary part is crucial for this to happen. More concretely, in this setup we have a total of three complex Kähler moduli, various complex structure moduli and the axio-dilaton. These last two get their masses through fluxes which implies

$$m_{cs}^2 \sim m_S^2 \sim g_s \frac{|W_0|^2}{\mathcal{V}^2}. \tag{3.45}$$

In the Kähler moduli sector a mass gap develops as a consequence of the assumed large volume stabilisation, with the blow-up mode τ_3 becoming heavier than the overall volume $\mathcal{V}$:

$$m_{\tau_3}^2 \sim g_s \frac{|W_0|^2}{\mathcal{V}^2} \qquad \gg \qquad m_{\mathcal{V}}^2 \sim g_s \frac{|W_0|^2}{\mathcal{V}^3}. \tag{3.46}$$

We therefore see that the volume modulus is, apart from the inflaton itself, the lightest field in the spectrum and so the one that would more easily be dynamical during the inflationary epoch. One must therefore make sure that

$$m_{\mathcal{V}}^2 \gg H^2 \sim V \tag{3.47}$$

such that no other field besides the fibre modulus τ_1 plays a role in inflation.

For the case when inflation takes place as the fibre modulus rolls to the left, the Hubble scale during the observable inflation is approximately

$$H^2 \sim \tilde{V}_0^L = V_0 \frac{C_1}{\beta_L} \sim g_s^{-2} \frac{W_0^6}{\mathcal{V}^4} \frac{\lambda^2 \Pi_1^2}{(C_1^{KK})^2} \tag{3.48}$$

and so pure single field dynamics requires

$$W_0^2 \frac{\lambda \Pi_1}{\beta_L} \sim W_0^2 \lambda \Pi_1 \ll \sqrt{g_s}, \tag{3.49}$$

where we have made use of the fact that in this regime $\beta_L = \mathcal{O}(1)$.

Noting that for inflation towards the right, the Hubble scale is

$$H^2 \sim \tilde{V}_0^R = V_0 \frac{C_2}{\beta_R} \sim g_s^{-2} \frac{W_0^6}{\mathcal{V}^4} \frac{\lambda^2 \Pi_2^2}{(C_2^{KK})^2}, \tag{3.50}$$

the requirement of single field dynamics leads to the following bound on the compactification parameters

$$\frac{W_0^2}{\mathcal{V}} \lambda \Pi_2 \ll \sqrt{g_s} \beta_R. \tag{3.51}$$

Recall that having all of the inflationary region in the geometric regime ($\tau_1 > 1$) implies that β_R is a small parameter: $\beta_R < 1/20$.

4   Inflationary Observables

Recall the effective inflationary potentials (3.30) and (3.42)

$$V_{inf}^R(\phi) = \tilde{V}_0^R \left(1 - e^{\frac{\kappa}{2}\phi}\right)^2, \quad V_{inf}^L = \tilde{V}_0^L \left(1 - e^{-\kappa\phi}\right)^2. \tag{4.1}$$

The predictions for a potential of type

$$V_{inf} = V_0 \left(1 - e^{\pm\nu\phi}\right)^2 \tag{4.2}$$

for the spectral index n_s and the tensor to scalar ratio r are well approximated by

$$n_s = 1 - \frac{2}{N}$$
$$r = \frac{1}{\nu^2}\frac{8}{N^2} + \mathcal{O}\left(\frac{1}{N^3}\right), \tag{4.3}$$

respectively, where N denotes the number of e-folds before the end of inflation and we have omitted sub-leading terms in $1/N$ for conciseness. The coefficient of the $\mathcal{O}(1/N^3)$ term can be obtained analytically but we omit its tedious and lengthy form here. Its numerical value differs between our two model classes of 'inflation to the left' and 'inflation to the right', which explains the split in the values for r in Table 1, where the first order phenomenological fingerprint is shown.

	$n_s(50)$	$n_s(60)$	$r(50)$	$r(60)$
right	0.960	0.967	0.0077	0.0055
left	0.960	0.967	0.0024	0.0016

Table 1. Numerical values for the spectral index n_s and the tensor-to-scalar ratio r calculated at 50 and 60 e-folds before the end of inflation.

Note how the predictions of inflating to the right are in line with [20]. When inflating to the left however, the tensor to scalar ratio decreases whereas the value of the spectral index remains universal. This is because for 'inflation to the right' the exponential term forming the inflationary plateau is the square-root of the analogous term when inflating to the left.

4.1   Higher Order Analysis

Recalling the scenarios $\mathcal{C}_2 > 0$, $\mathcal{C}_1 < 0$ or $\mathcal{C}_1 > 0$, $\mathcal{C}_2 < 0$, we see that the resulting inflationary potentials feature an inflection point along the inflationary trajectory. We now turn to study possible higher order corrections to the inflationary observables due to the presence of this inflection point.

Consider scenario $\mathcal{C}_2 > 0$, $\mathcal{C}_1 < 0$ and the inflationary potential with the $\mathcal{C}_1$-term included

$$V_{inf}^R \sim V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} - \mathcal{C}_2\sqrt{\tau_1} + \mathcal{C}_2^{loop}\tau_1\right). \tag{4.4}$$

The inflaton inflates to the right while the $\mathcal{C}_1$-term induces the inflection point. Note that we consider the $\mathcal{C}_1$-term arising from $V_{(1)}$ to be of importance while we have still omitted the string loop induced $\mathcal{C}_1^{loop}$-term. The reasoning is that first the string loop term is τ_1^{-1} suppressed with regard to the higher derivative term. Second, for typical values of C_1^{KK} as given in (2.6), the C_1^{loop}-term will be additionally suppressed. At last, while the string loop term scales with g_s, the higher derivative term is $g_s^{-3/2}$ enhanced.

The canonically normalised and uplifted potential hence receives a rising correction at large e-foldings, i.e.

$$V_{inf}^R(\phi) \sim \tilde{V}_0^R \left(1 - 2e^{\frac{\kappa}{2}\phi} + \varepsilon^2 e^{-\kappa\phi}\right), \quad \varepsilon^2 = \frac{|\mathcal{C}_1|}{4\,\mathcal{C}_2}\beta_R^3 \tag{4.5}$$

where we have already expanded the inflationary potential and have omitted the string loop induced term for notational ease.[4] From the above we may already infer that $\varepsilon^2 \ll 1$ for the inflationary plateau not to be spoiled. The slow-roll parameters then receive ε^2 dependent corrections of the form

$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{1}{2}\left(-\kappa\,e^{\frac{\kappa}{2}\phi} - \kappa\,\varepsilon^2 e^{-\kappa\phi}\right) \tag{4.6}$$

$$\eta_V = \frac{V''}{V} = -\frac{1}{2}\kappa^2 e^{\frac{\kappa}{2}\phi} + \kappa^2\varepsilon^2 e^{-\kappa\phi}, \tag{4.7}$$

where we have taken the potential to be slowly varying during inflation, i.e. $V_{inf}^R \sim const$. Recalling $dN = (2\epsilon_V)^{-1/2}d\phi$, i.e.

$$N \sim 2\kappa^{-2}e^{-\frac{\kappa}{2}\phi} + \mathcal{O}(\varepsilon^2) \tag{4.8}$$

we hence arrive at the expression for the spectral index n_s including higher order corrections

$$n_s = 1 - \frac{2}{N} - 3\,\varepsilon^2\kappa^4 N + \frac{\varepsilon^2\kappa^6}{2}N^2 + \dots \tag{4.9}$$

The above suggests that there is a further phenomenological fingerprint in the form of running of the spectral index which may manifest itself as a loss of power in the CMB temperature power spectrum at large angular scales corresponding to the onset of observable e-folds. Following the analysis of [19], we recall that $n_s = d\ln P/d\ln k = P^{-1}dP/dN$ and hence

$$\frac{\Delta P(\delta n_s)}{P}\bigg|_{N+\Delta N}^{N} = \int_{N+\Delta N}^{N} \delta n_s \sim \delta n_s\,\Delta N. \tag{4.10}$$

In the case of the inflaton inflating to the right, we found above that

$$\delta n_s = -3\,\varepsilon^2\kappa^4 N + \frac{\varepsilon^2\kappa^6}{2}N^2. \tag{4.11}$$

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[4]During inflation, the term ensuring the existence of the minimum is negligible.

Taking $N = 55$ and requiring $\delta n_s \lesssim 0.008$, which is the 2-$\sigma$ range for the n_s measurement from Planck, we find an upper bound on ε^2 to be [5]

$$\varepsilon^2 = \frac{|\mathcal{C}_1|}{4\,\mathcal{C}_2}\beta_R^3 \lesssim 2.4 \times 10^{-6}. \tag{4.12}$$

Thus, we may now evaluate (4.10) with the upper bound on ε^2 and the range of e-folds $\Delta N \simeq 5$ over which power-loss is usually considered to occur. We find the power loss to be

$$\left.\frac{\Delta P(\delta n_s)}{P}\right|_{N+\Delta N}^{N} \simeq -0.04 \tag{4.13}$$

which means a suppression of 4%. This is in agreement with earlier works employing exponentially rising corrections to the inflationary plateau [35–43].[6]

Note how the bound on ε^2 also restricts

$$\frac{|\mathcal{C}_1|}{4\,\mathcal{C}_2}\beta_R^3 \sim \lambda^{-3}\,\mathcal{V}\,g_s^{15/2}\,\Pi_1\,\Pi_2^{-4}(C_2^{KK})^6 \lesssim 2.4 \times 10^{-6} \tag{4.14}$$

and hence gives a constraint that has to be fulfilled in the first place when considering observationally viable inflation to the right.

Considering the scenario where inflation occurs for the inflaton rolling to the left with $\mathcal{C}_1 > 0\,,\mathcal{C}_2 < 0$, we now start with the potential

$$V_{inf}^{L} \sim V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} + \frac{\mathcal{C}_1^{loop}}{\tau_1^2} + \mathcal{C}_2\sqrt{\tau_1} \right). \tag{4.15}$$

The $\mathcal{C}_2$-term induces the inflection point. Again, we have omitted the string loop induced term as it is suppressed with regard to the higher derivative $\mathcal{C}_2$ term by similar reasoning as was employed when justifying the omission of the $\mathcal{C}_1^{loop}$-term when studying inflation to the right. The canonically normalised and uplifted potential hence receives a rising correction at large e-foldings, i.e.

$$V_{inf}^{L}(\phi) \sim \tilde{V}_0^L \left(1 - 2e^{-\kappa\phi} + \varepsilon^2 e^{\frac{\kappa}{2}\phi} \right), \quad \varepsilon^2 = \frac{|\mathcal{C}_2|}{\sqrt{2}\,\mathcal{C}_1}\beta_L^{3/2} \tag{4.16}$$

where we have again expanded the inflationary potential and have omitted the string loop induced term as it plays no role on the inflationary plateau. Similarly to the

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[5]Note that for $\mathcal{V} \sim 10^3$ and $\Pi_1 \sim \Pi_2$ the below bound requires $\beta_R \lesssim 10^{-3}$, thus, placing a stronger constraint on β_R than the minimal required length of the plateau.

[6]The comparison of our estimate here with the result in [38] may at first looking conflicting. However, [38] wished to ensure a multipole dependence of δn_s such that $\delta n_s > 0$ for a drastic blue shift for $\ell < \mathcal{O}(20)$ and very suddenly reaching $\delta n_s = 0$ at $\ell > \mathcal{O}(20)$. For such a suddenly shutting down power suppression a much steeper rising exponential is necessary than we get in our models here, as shown in [38]. However, the fit to the data is already consistently improved with a small power loss of $\lesssim 4\%$ which is very slowly shutting down as a function of increasing ℓ. Such a slowly varying power-loss can arise from more slowly rising exponentials with $\mathcal{O}(1)$ coefficients in the exponent, as arising in our models here, and hence explaining the difference to [38].

rolling to the right case above, we can derive n_s and get

$$n_s = 1 - \frac{2}{N} - \frac{3\sqrt{2}\delta^2\sqrt{\kappa^2 N}}{N} + \frac{\delta^2\kappa^2\sqrt{\kappa^2 N}}{\sqrt{2}} - \frac{1}{2}3\delta^4\kappa^4 N + \dots \quad . \qquad (4.17)$$

Considering the 2-σ bounds by PLANCK, i.e. requiring $\delta n_s \lesssim 0.008$ at $N = 55$, we obtain the upper bound

$$\varepsilon^2 \sim \lambda^{-3/2} \mathcal{V}^{-1} \left(g_s^{5/2}\mathcal{V}\right)^{3/2} \Pi_2 \Pi_1^{-5/2}(C_1^{KK})^{3/2} \lesssim 10^{-3} \, . \qquad (4.18)$$

Saturation of that bound gives again rise to a suppression of power of about 4%. Note that the above analysis may also readily be done for both $\mathcal{C}_i > 0$ translating to $\varepsilon^2 < 0$. The requirement of a sufficiently flat plateau, i.e. a constraint on the running of n_s, remains unchanged and hence similar bounds are obtained.

Given the bounds on ε induced by the data-compatible range of δn_s, we also find that the contribution of the δn_s to the magnitude of running $dn_s/d\ln k = -dn_s/dN$ is typically $\lesssim \mathcal{O}(10^{-4})$ and hence at least an order of magnitude smaller than the contribution to the running from $n_s^{(0)} - 1 = -2/N$ which is about $dn_s^{(0)}/d\ln k \sim -10^{-3}$.

4.2 Examples

In this section we present explicit examples in order to illustrate the points made above on naturalness and tuning of the models under discussion. In table 4.2 we put forth two sets of parameters for each regime. We assume $|\lambda| = 10^{-3}$ throughout, in accordance with the estimate of Eq. (2.10).

	W_0	g_s	$\mathcal{V}$	τ_1^{min}	Π_1	Π_2	C_1^{KK}	C_2^{KK}	n_s	r
$\mathcal{R}_1$	5	0.2	625.5	3000	0	100	0.00242	0.799	0.968	0.0067
$\mathcal{R}_2$	25	0.3	1886.2	3500	0	10	0.000859	0.732	0.967	0.0066
$\mathcal{L}_1$	2	0.3	460	3	100	1	0.163	0.0288	0.966	0.0018
$\mathcal{L}_2$	5	0.4	1031.6	6	50	0	0.189	0.0266	0.969	0.0021

Table 2. Examples of compactifications parameters and inflationary observables for inflation to the left ($\mathcal{L}_1$ and $\mathcal{L}_2$) and to the right ($\mathcal{R}_1$ and $\mathcal{R}_2$). Inflationary observables are computed at $N_e = 55$.

Let us start by discussing some generic features of the inflation to the right regime, corresponding to examples $\mathcal{R}_1$ and $\mathcal{R}_2$. In this regime, control over the l_s expansion in the whole of the inflationary range leads us to consider large τ_1^{min}. More concretely, the field excursion between the minimum and the 55 efoldings point is $\Delta\phi = 6.46$, which taking into account the canonical normalisation of Eq. (3.11), implies $\tau_1^{min} \geq 1700$. The overall volume $\mathcal{V}$ is constrained by the normalisation of the density perturbations to be at most $\mathcal{O}(10^3)$. The combination of these two constraints leads to anisotropic compactification manifolds, with a large fibre and a relatively small base, with $\tau_2^{min} \sim \mathcal{O}(10)$. Another stand out feature of this regime is the sensitivity to the topological

integer Π_1. Due to fact that Π_1 enters in V multiplied by the overall volume, if one considers manifolds with $\Pi_1 \neq 0$ the $\mathcal{C}_1$ generated exponential tends to be relevant in the inflationary regime, leading at the very least to deviation from the first order observables of section 4 and in more severe cases to the destruction of the inflationary plateau. This situation can be avoided if one considers non-zero Π_1 and a very large $\Pi_2 \sim \mathcal{O}(10^4)$ or alternatively if one considers a manifold with intersection numbers such that $\lambda_1 \ll 1$. Note that Calabi-Yau threefolds with either $\Pi_1 = 0$ or $\Pi_2 = 0$ can exist. Since we did not choose Kähler cone variables, these statement translate into the fact that a particular linear combination of Π'_i vanishes, where Π'_i encode the topological information of the second Chern class with respect to a basis of cycles spanning the Kähler cone as defined in (2.12).

Turning our attention to the inflation to the left regime illustrated by examples $\mathcal{L}_1$ and $\mathcal{L}_2$, we note that these correspond to manifolds with a large base and a relatively small fibre (though examples with larger fibre can be found). Since the exponential generating the inflationary plateau has a coefficient that is volume enhanced with respect to the one that steepens the plateau at large ϕ, one can find points in parameter space that lead to a viable inflationary regime where neither of the topological integers vanishes, while keeping the topological integers in their natural range $\Pi_i \leq \mathcal{O}(10^3)$. Once again, the requirement of a mass hierarchy and of the correct normalisation for the amplitude for the density perturbations constrains the overall volume to be $\mathcal{V} \sim \mathcal{O}(10^2 - 10^3)$. We also note that in accordance to the estimate of Eq. (2.6), the complex structure dependent coefficients are suitably small: $C_{1,2}^{KK} < 1$ in both right and left inflationary regimes.

All in all, these explicit examples show that the inflationary models built in this work are easily obtained in the perturbative regime of both α' and g_s expansions of type IIB orientifold compactifications, without a need for just moderate tuning of the underlying microscopic parameters.

5 Discussion

Knowledge of the string effective action is ever evolving and to date incomplete. Whenever a new work comes about shedding new light into the form of the action, the phenomenologist must investigate what are the consequences of the novel structures and terms to the history and evolution of the Universe. In this work we have carried out an analysis in this spirit, investigating the consequences of the higher derivative corrections to the action derived in [25] for cosmic inflation in the context of type IIB string compactifications.

In general terms, the new higher derivative generated terms in the potential are suppressed since they scale as $\mathcal{V}^{-4}$. This renders them harmless and of limited use whenever other larger terms are present. Such terms will therefore have no impact in the Kähler moduli stabilisation in the context of LVS on compactification geometries with Swiss-Cheese structure and in the context of KKLT. However, large volume scenarios with different compactification geometries exist in which, through symme-

tries and cancellations, such terms are the leading ones and are therefore crucial for phenomenology.

One such setup is analysed here: the stabilisation of the fibre modulus in K3-fibered CY manifolds in the context of LVS compactifications and its inflationary phenomenology. In the absence of non-perturbative effects supported on the fibre modulus, this Kähler modulus is flat at leading order in g_s and to third order in α'. Previous works have focused on generating a potential for τ_1 via higher order terms in the g_s expansion [20] or via the introduction of non-perturbative effects like poly-instantons [21]. In this study the potential is generated by the interplay between the higher derivative terms and the string loop corrections to the action.

Though the higher derivative corrections can by themselves stabilise the fibre modulus, we have shown that the resulting inflationary phenomenology is in tension with the CMB observations. By combining them with the string loop corrections one can simultaneously generate a controlled minimum and an inflationary plateau. Depending on the underlying microscopic parameters the current model features two regimes which we dubbed inflating to the right and to the left. Both are of Starobinsky-like form yielding $n_s \sim 0.97$ while differing in the tensor fraction: for inflation to the right (left) one finds $r \simeq 0.007$ ($r \simeq 0.002$).

In our analysis we have not only included the contributions giving rise to the inflationary plateau but also some potentially dangerous terms, whose effect is to spoil the flatness of the inflationary region. We have shown that though not guaranteed at all points in moduli/parameter space, these terms can be rendered harmless for moderately tuned choices of the underlying microscopic parameters. Effectively, the smallish-g_s based decoupling of the string loops from the inflationary plateau, and the large-volume control of successively higher-order α'-corrections constitutes a leftover from the approximate Kähler moduli shift symmetry at parametrically large volume. However, the leftover is imperfect leading to the still required moderate level of tuning for the string loop coefficients. We have furthermore shown that in some cases such unwanted terms may give rise to a percent level power loss at the level of the primordial scalar power spectrum for which there are some hints in the data.

As a final observation we mention that exponentially flat plateau potentials might also arise from the 3-form flux induced complex structure moduli potential. There, canonically normalizing the kinetic term can produce exponential field dependence in the potential similar to the case of the fibre Kähler modulus here. Since in that case the scalar potential would arise at string tree level *and* $\mathcal{O}(\alpha'^0)$ this might improve the level of protection from higher-order corrections.

Acknowledgments

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